Defects and shell stability under external pressure

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For any triangulation of a sphere:

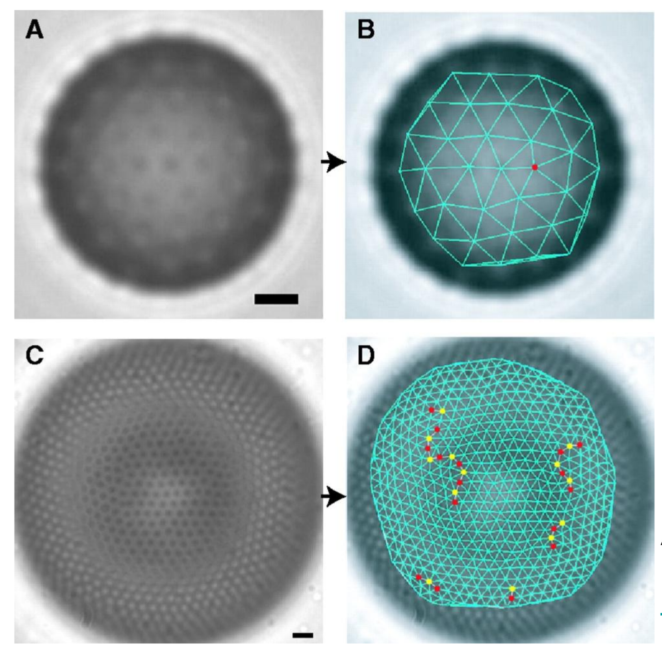
$$\sum_i q_i = \sum_i (6 - c_i) = 6\chi = 12$$

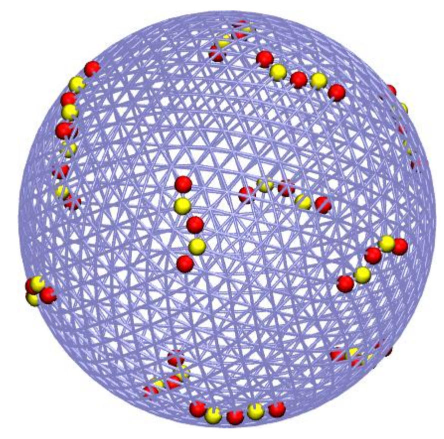
If limited to $q = \pm 1$ charges:

Minimal set of defects: Twelve +1 disclinations

Extended defect arrays: Grain boundary "scars"

Grain boundary scars



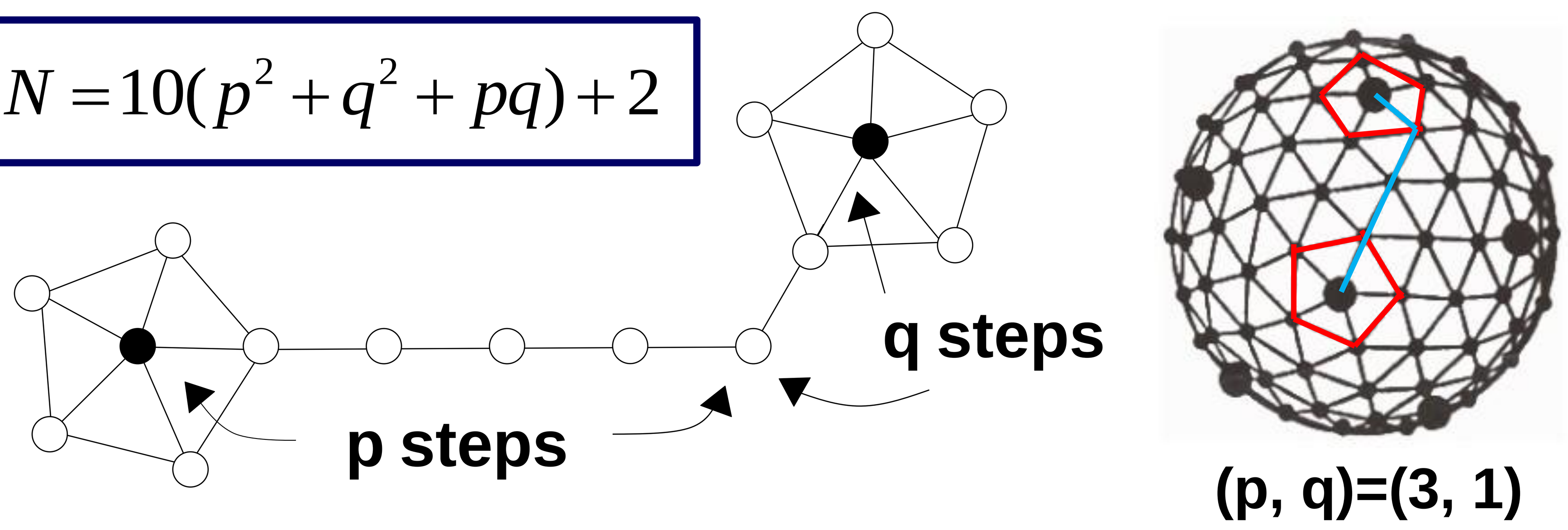


Red: fivefold
Yellow: sevenfold

A. R. Bausch *et al*. *Science*, (2003).
http://thomson.phy.syr.edu.thomsonapplet.htm

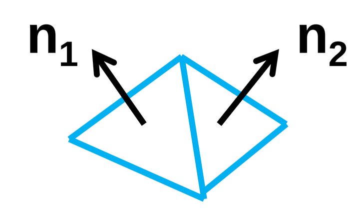
Icosadeltahedral shells and Caspar-Klug notations

$$N = 10(p^2 + q^2 + pq) + 2$$



D. L. D. Caspar and A. Klug. *Cold Spring Harb. Symp. Quant. Biol, (1962).*
J. Lidmar, L. Mirny and D. R. Nelson. *PRE*, (2003).

Energy



$$F_{tot} = F_{stretching} + F_{bending} + PV$$

Continuum elasticity:

$$F_s = \frac{1}{2}\int dS\left(2\mu u_{ij}^2 + \lambda u_{kk}^2\right)$$

$$F_b = \frac{1}{2}\int dS\left(2\kappa H^2 + \kappa_G K\right)$$

Discretized version:

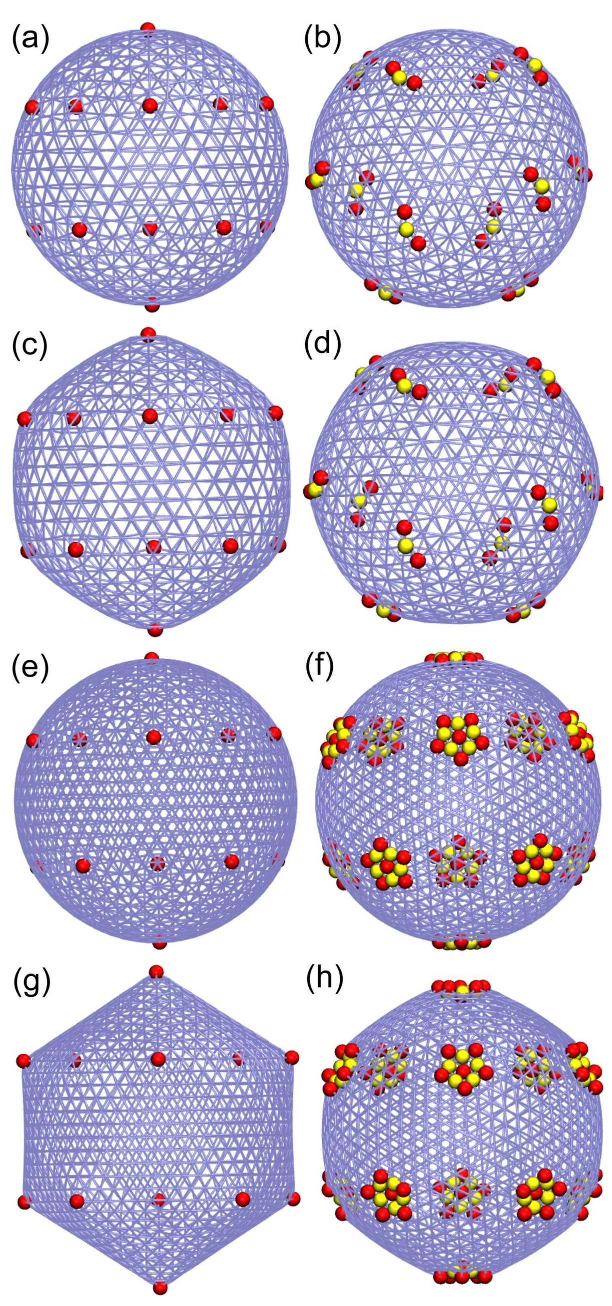
$$F_s = \frac{\varepsilon}{2}\sum_{\langle ij\rangle}\left(|\mathbf{r}_i - \mathbf{r}_j| - a\right)^2$$

$$F_b = \frac{\tilde{\kappa}}{2}\sum_{\langle IJ\rangle}\left(\hat{\mathbf{n}}_I - \hat{\mathbf{n}}_J\right)^2$$

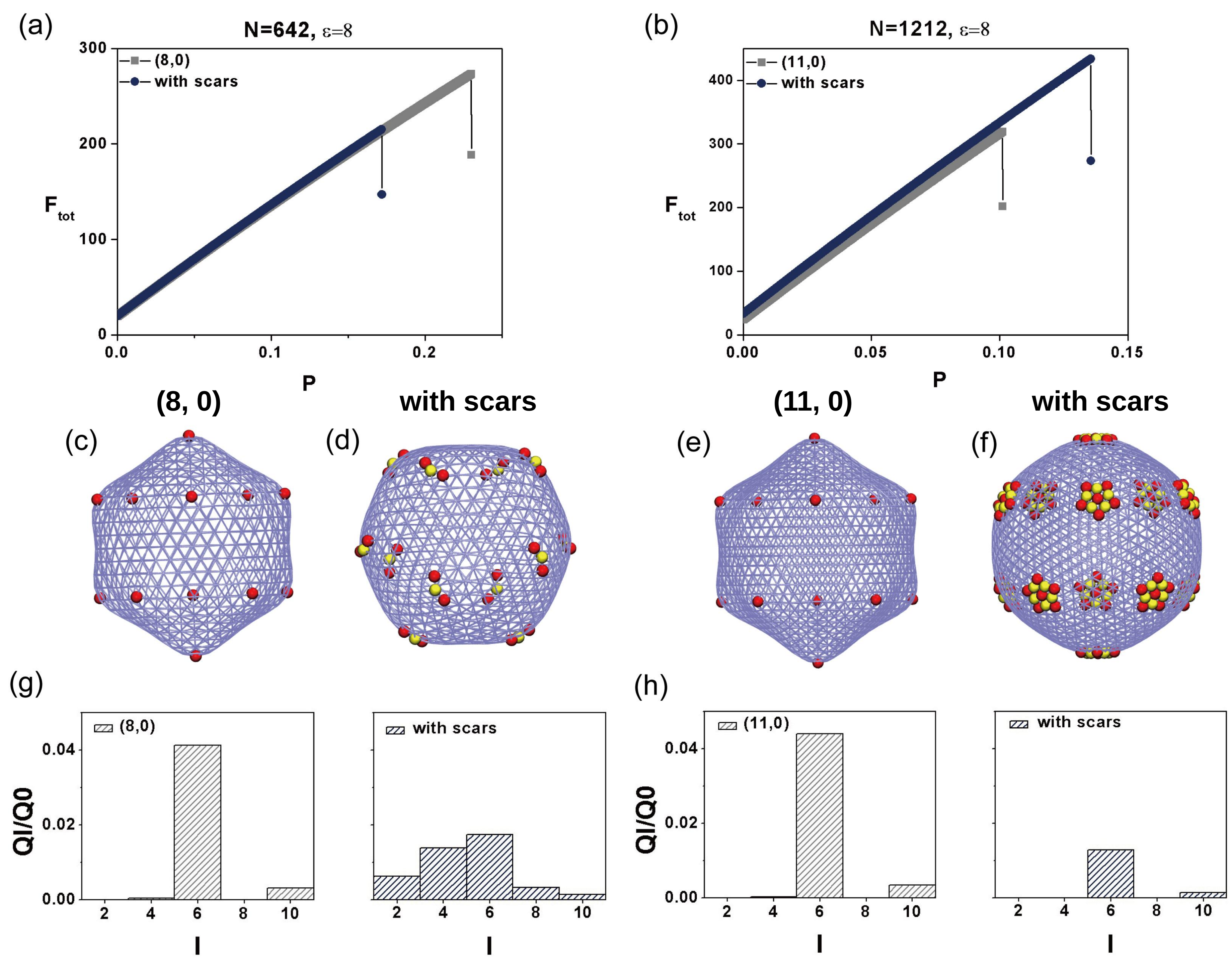
L. D. Landau and E. M. Lifshitz. *Theory of Elasticity, 3rd edition*, (2003).

Comparison of shells without and with scars in (8,0) and (11,0) cases

In the absence of pressure

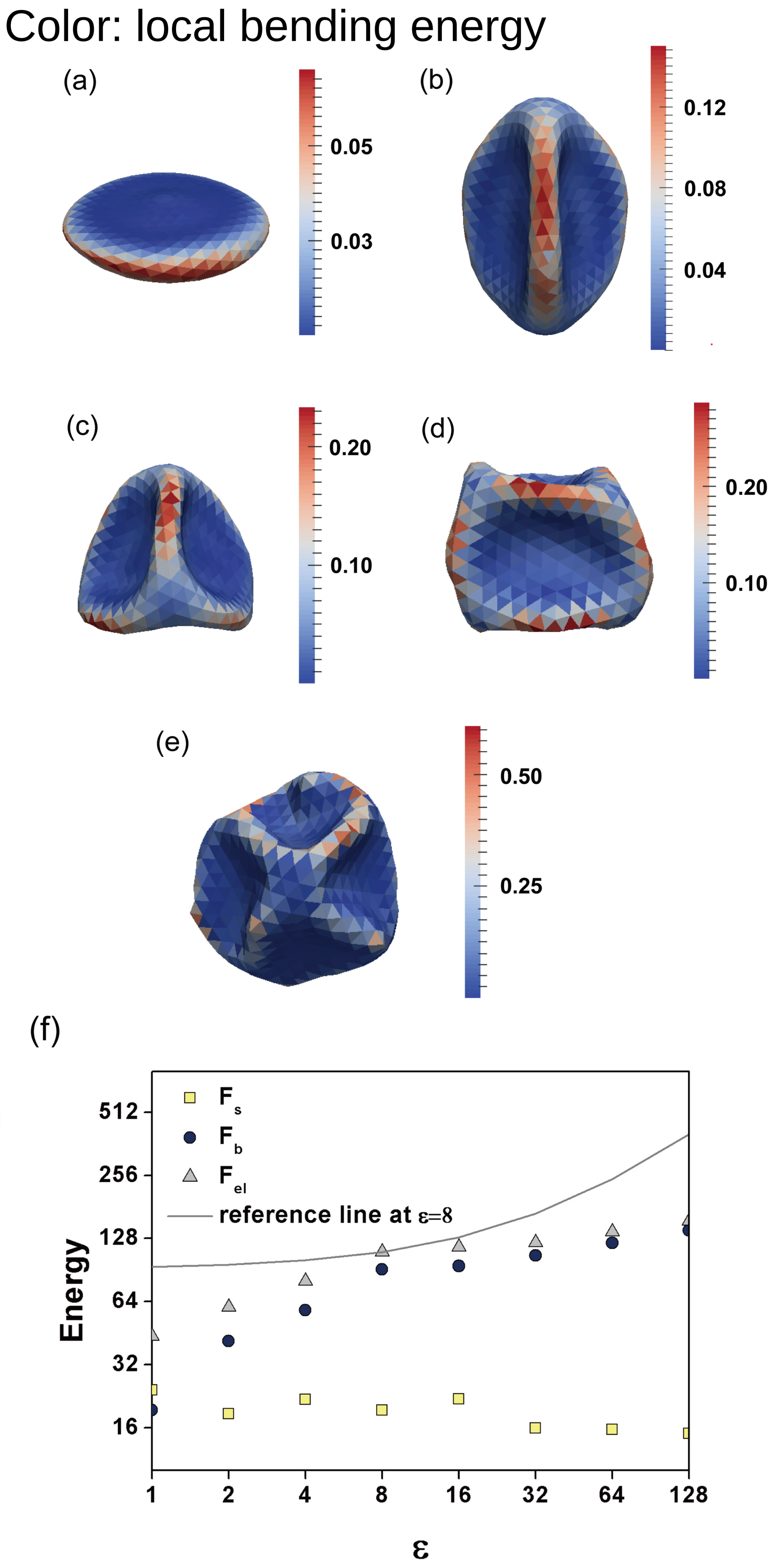


Configurations before collapse

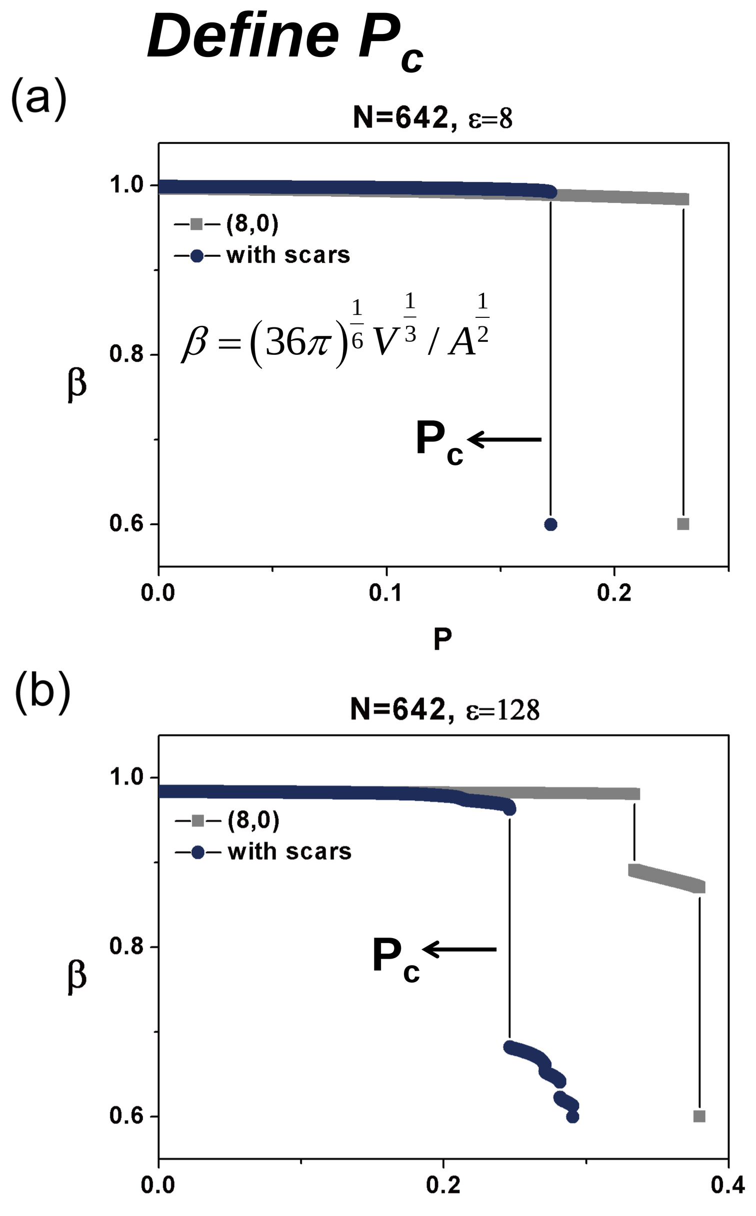


Configurations during collapse

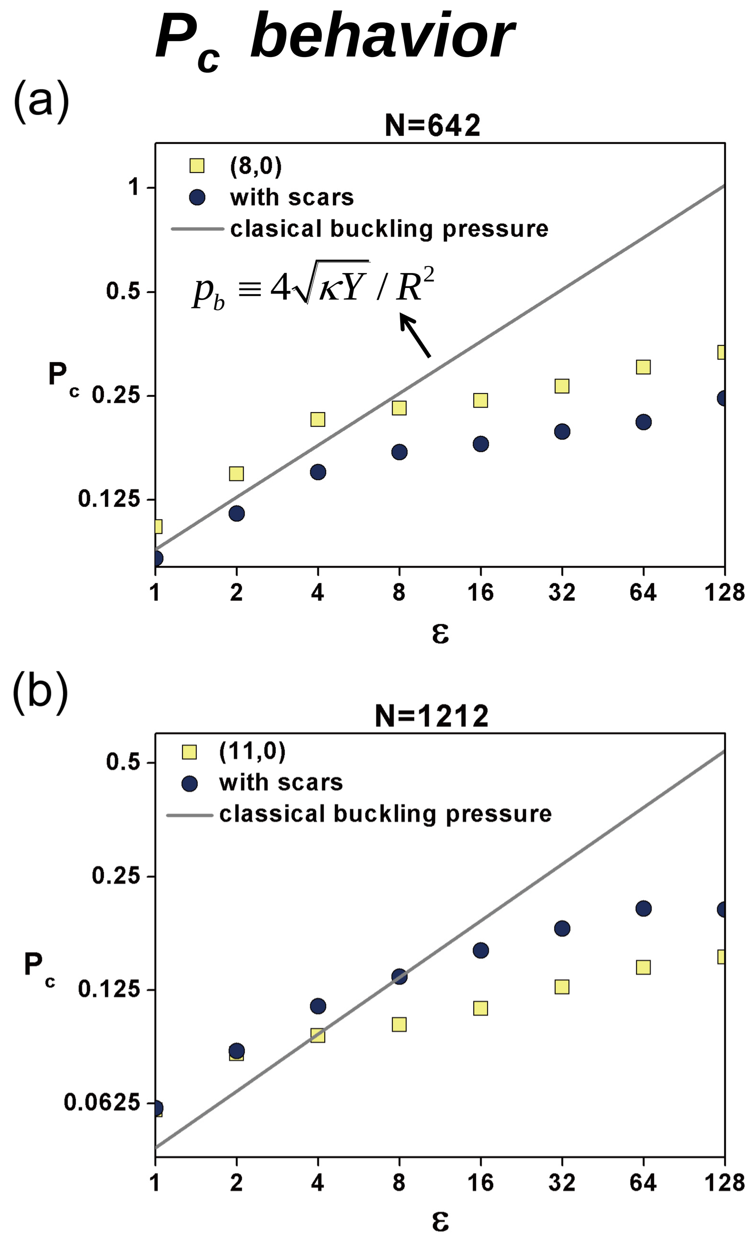
Color: local bending energy



Define P_c



P_c behavior



l=6, 10 components are a signal of icosahedral symmetry.

Conclusions

Mechanisms affect the critical pressure:
(1) the preservation of icosahedral symmetry
(2) the area occupied by scars

Acknowledgments
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